

BACKGROUND ART

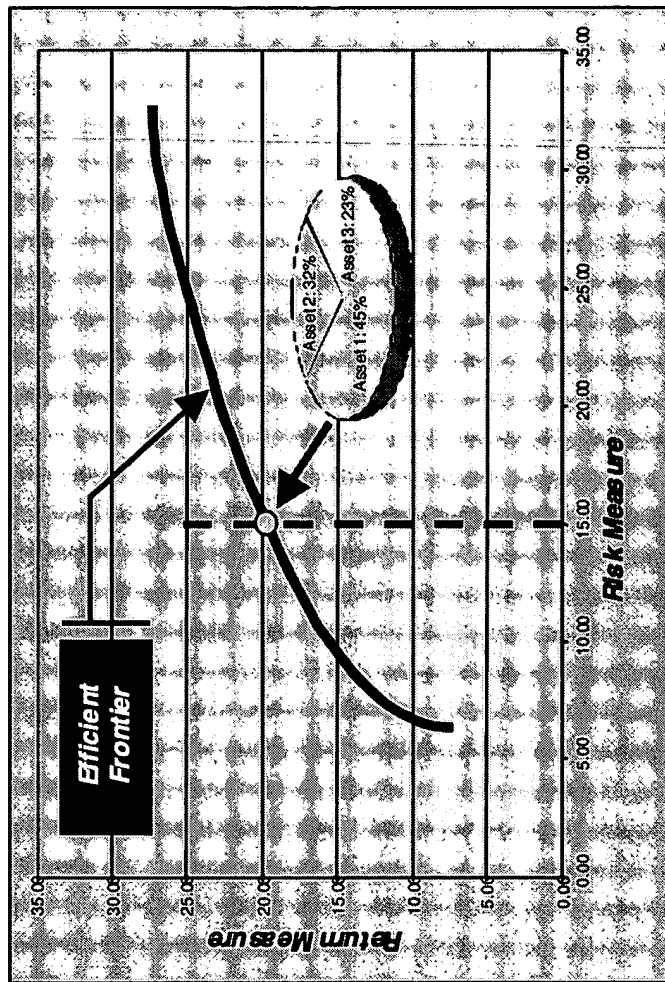


Fig. 1

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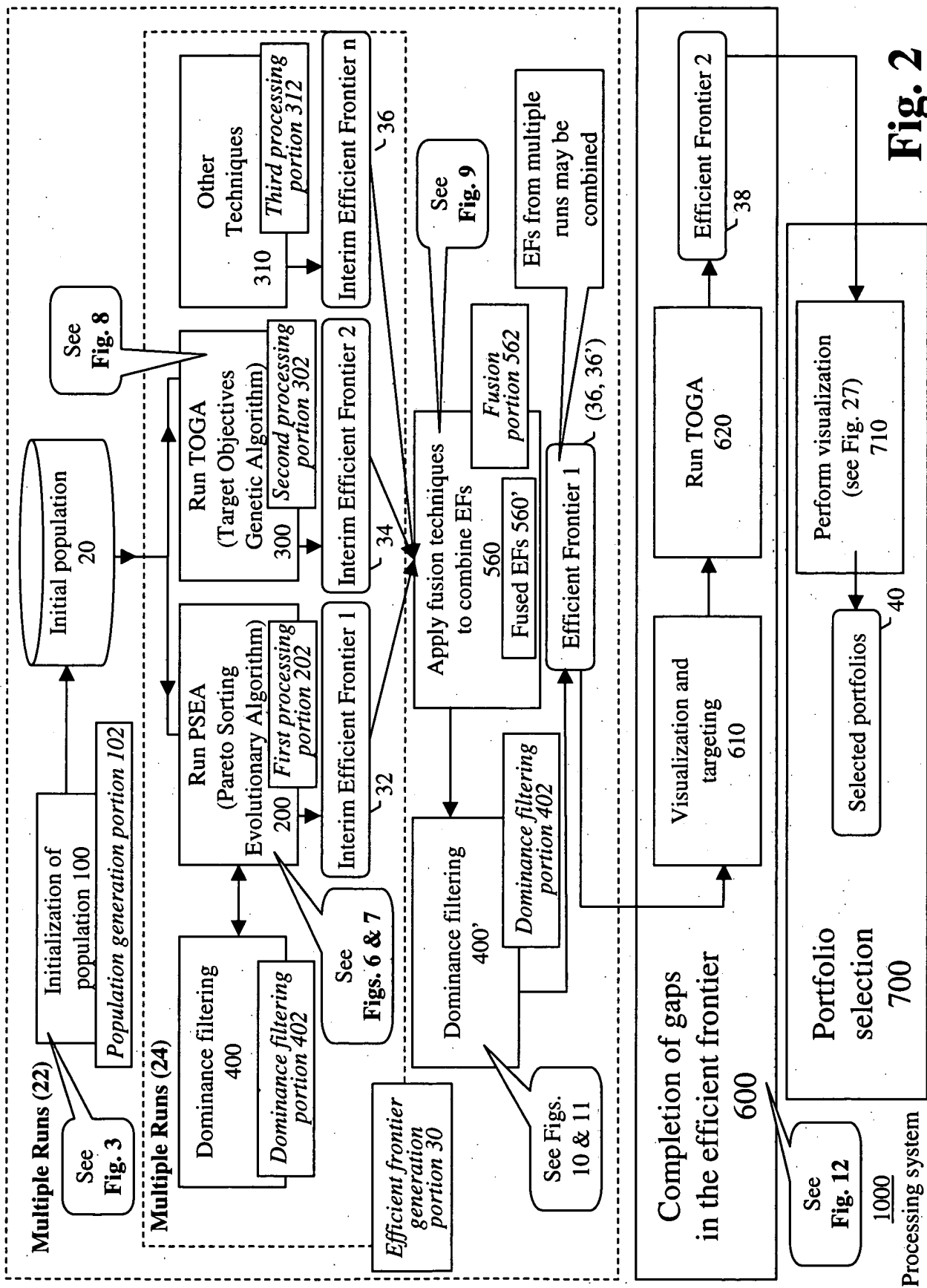


Fig. 2

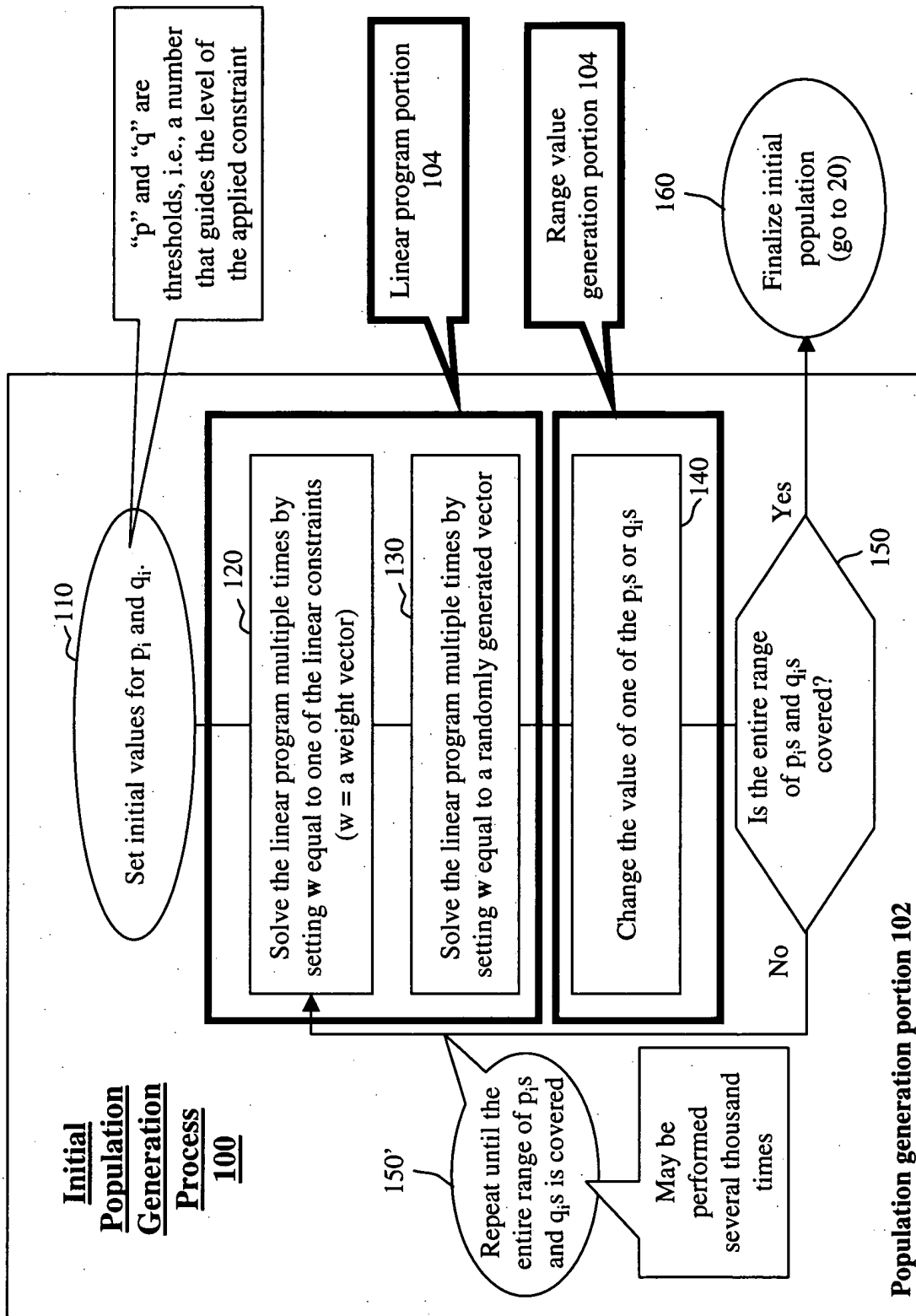


Fig. 3

Fig. 4

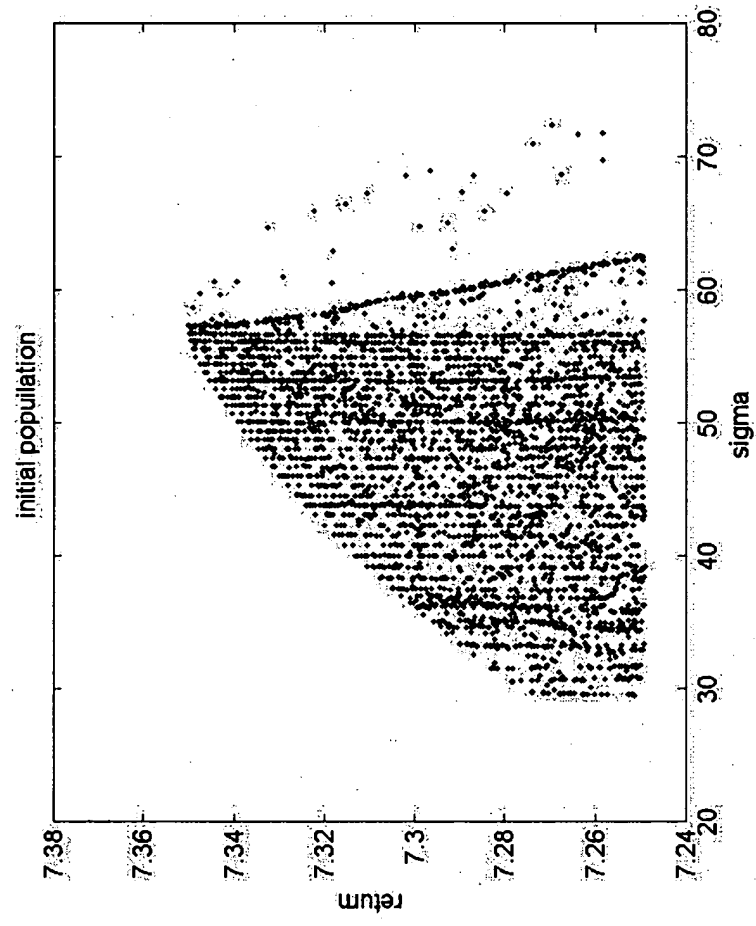
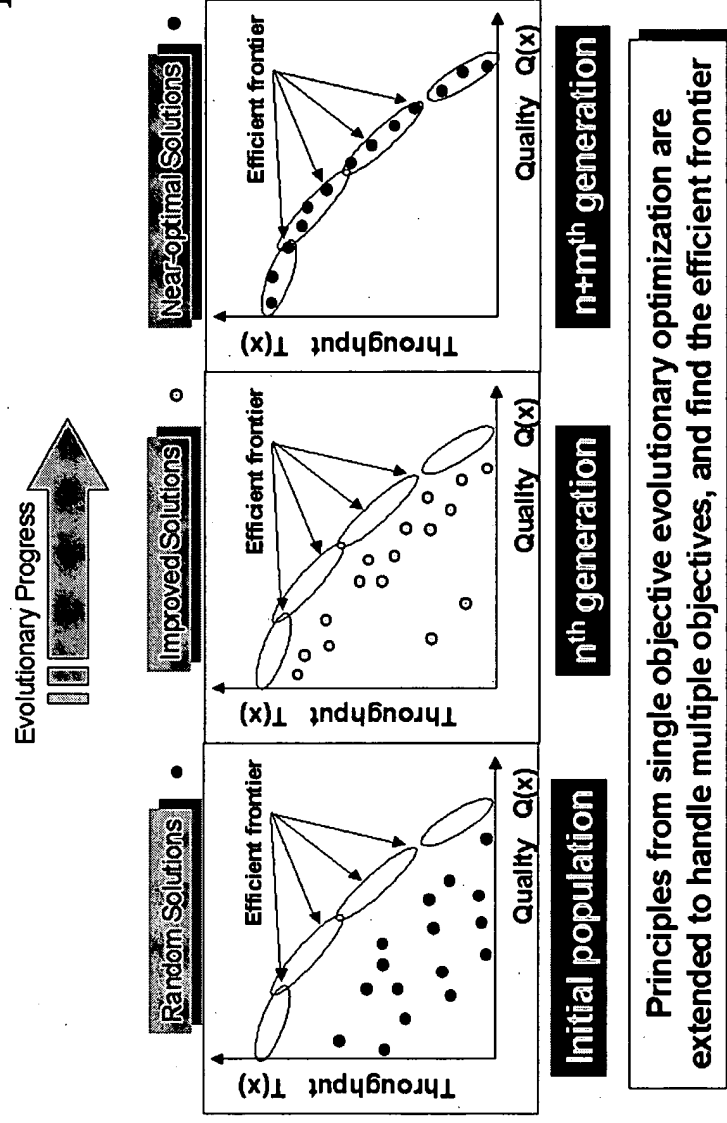


Fig. 5



Pareto Sorting Evolutionary Algorithm (PSEA)



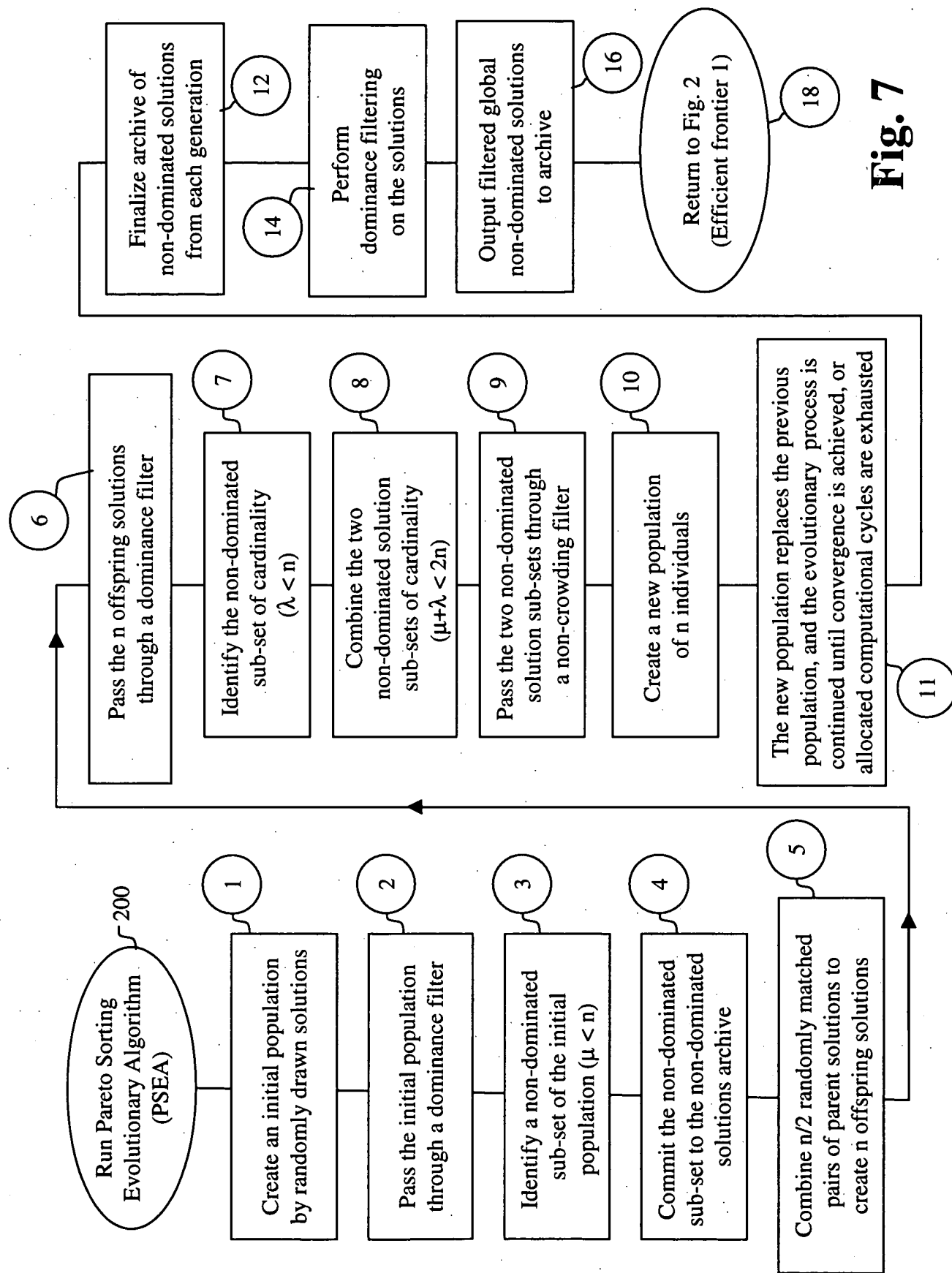
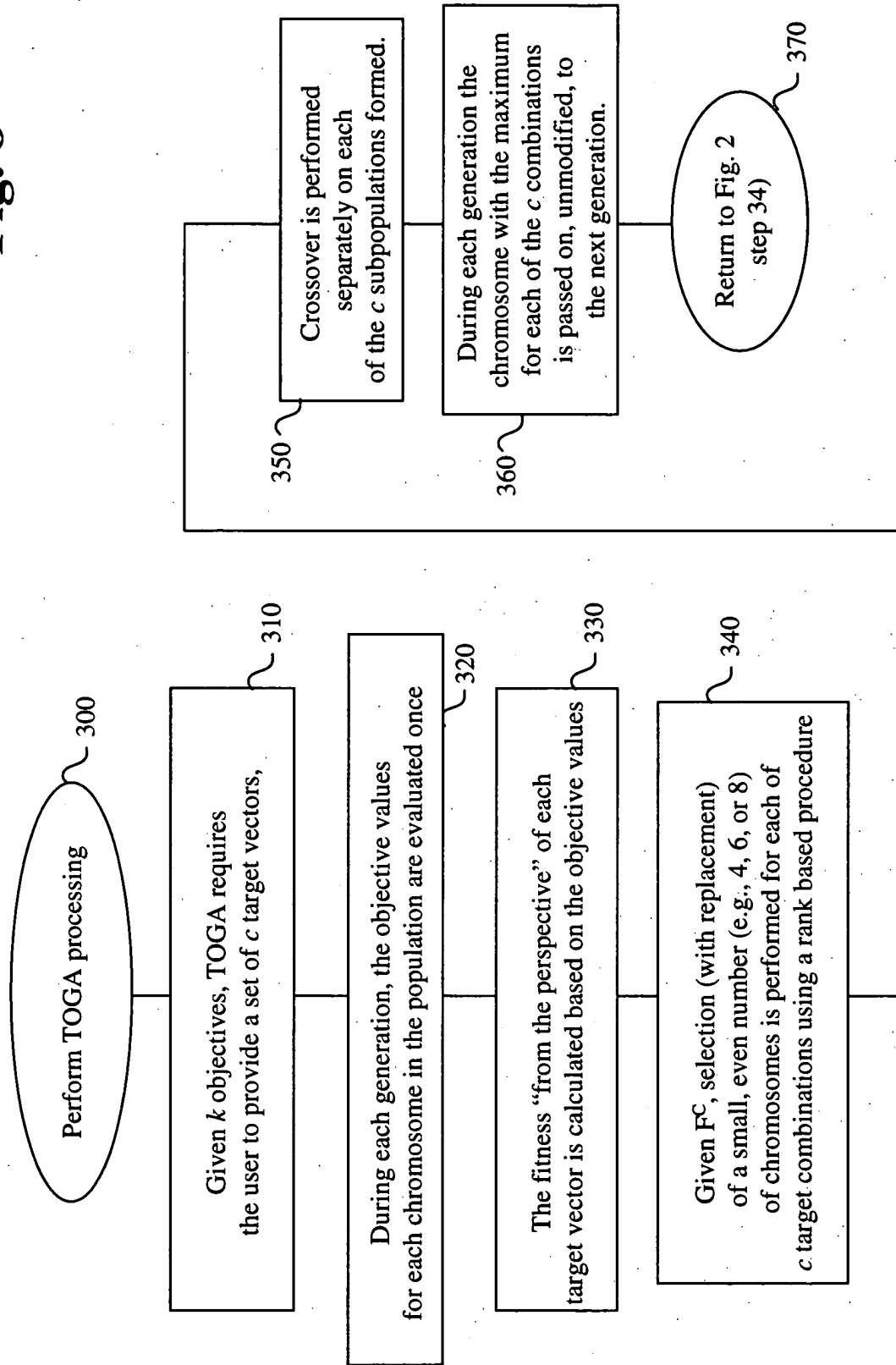


Fig. 7

Fig. 8



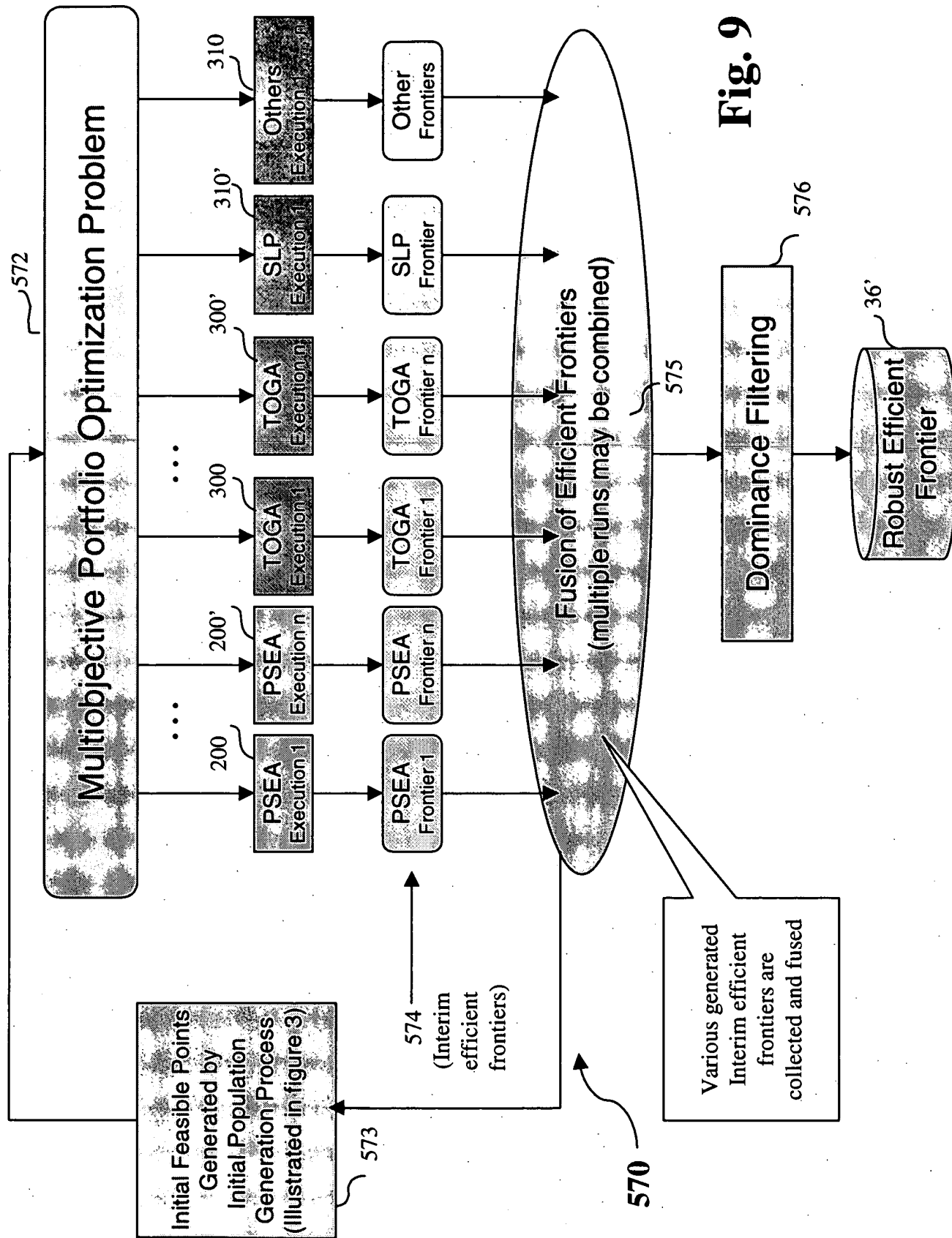
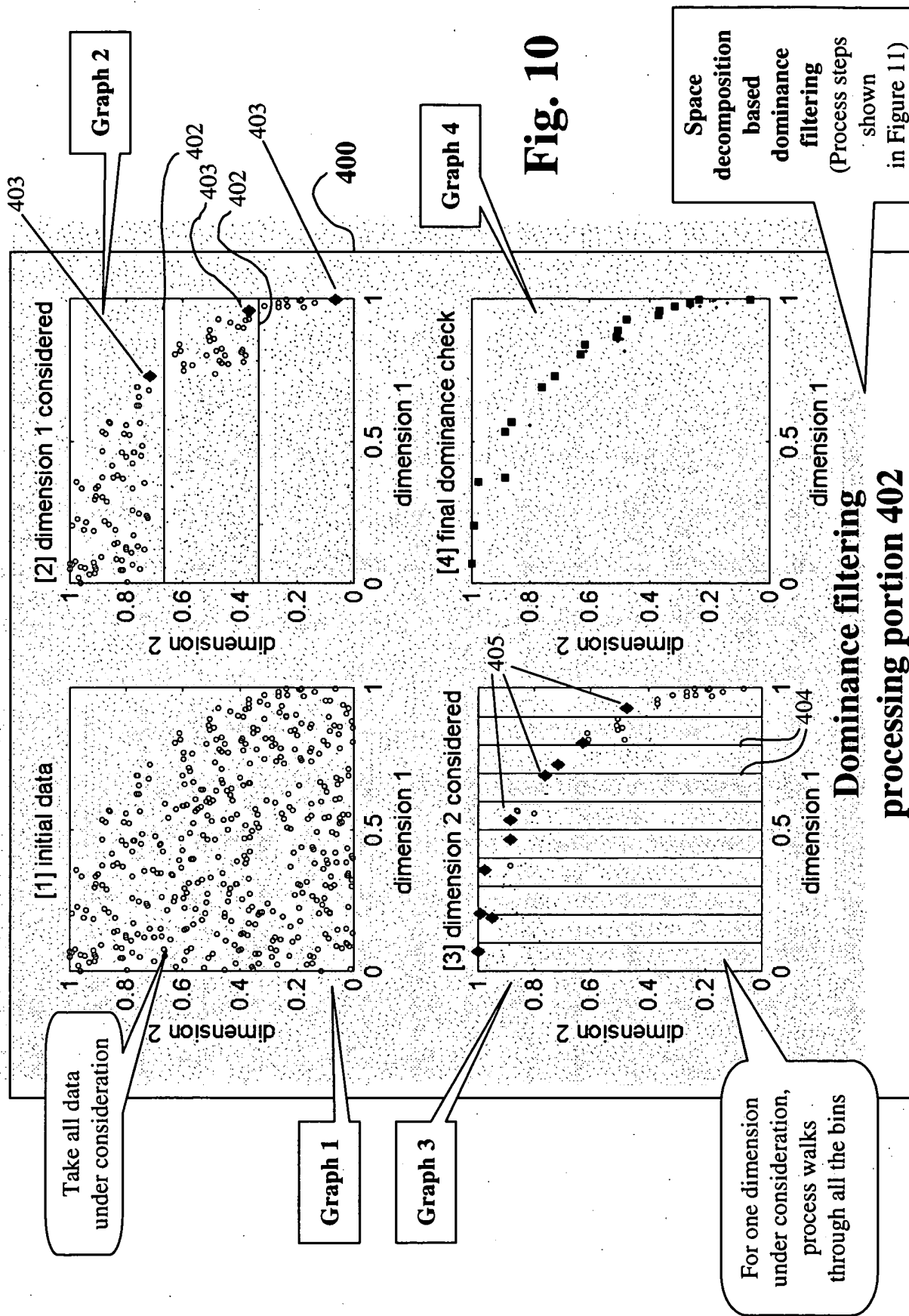


Fig. 9



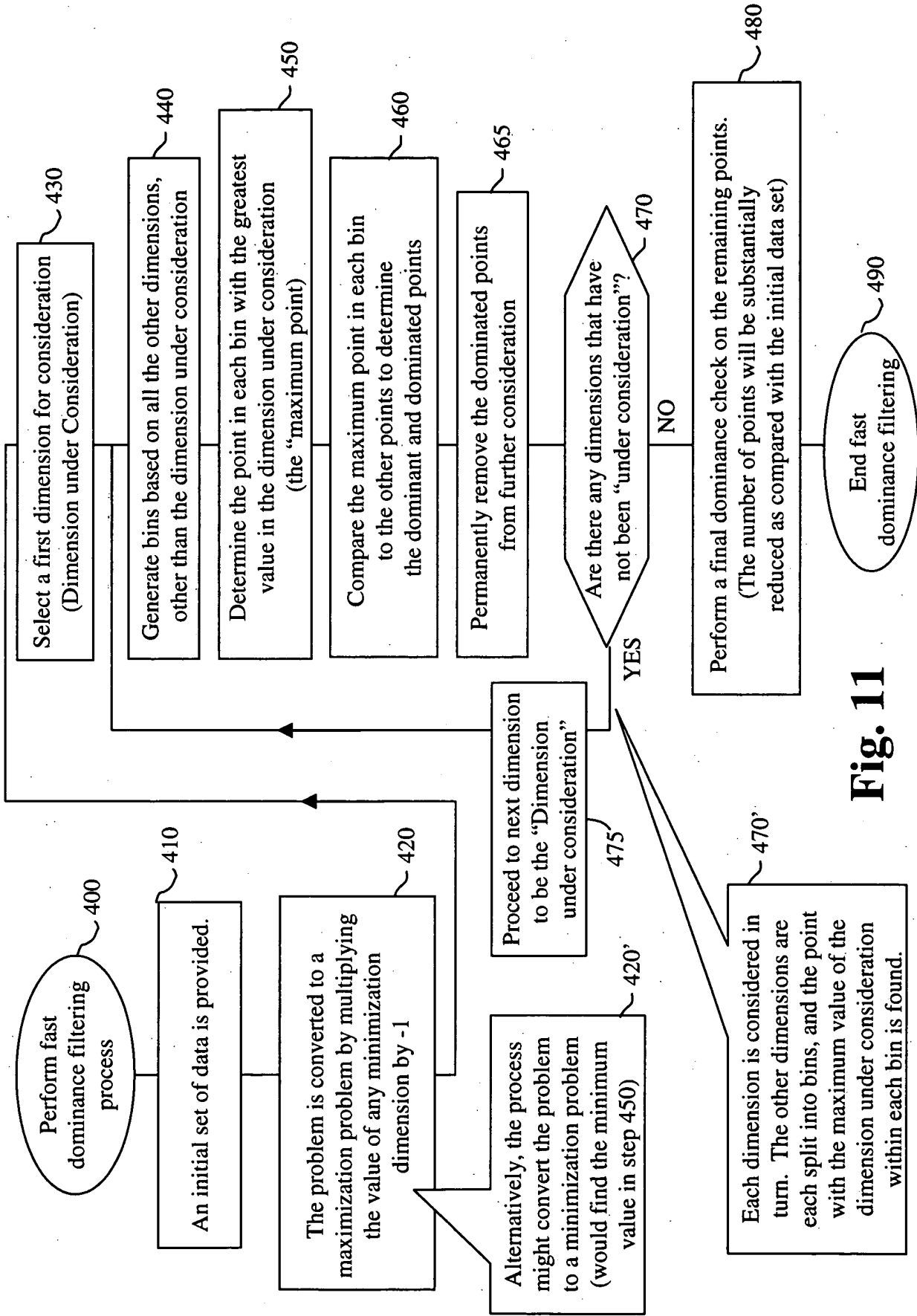


Fig. 11

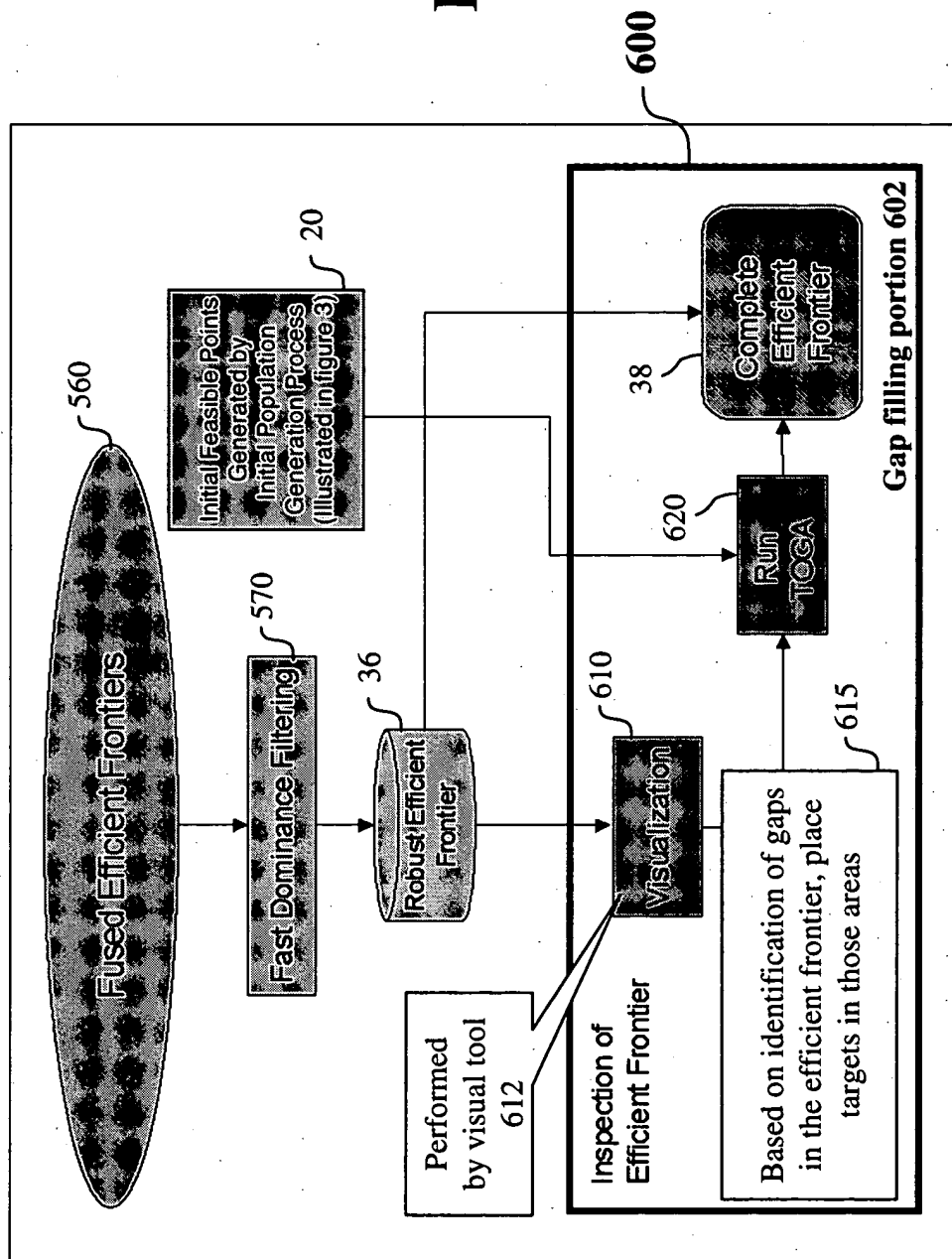
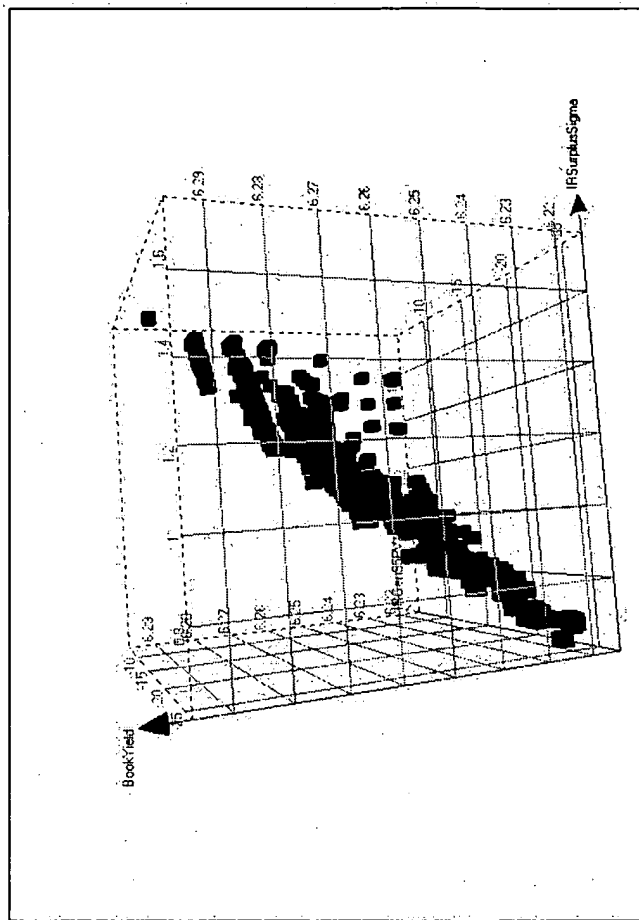


Fig. 12

Process to interactively fill any gaps in the identified efficient frontier

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Fig. 13



Efficient Frontier in a 3D View

Example of Parallel coordinate plot

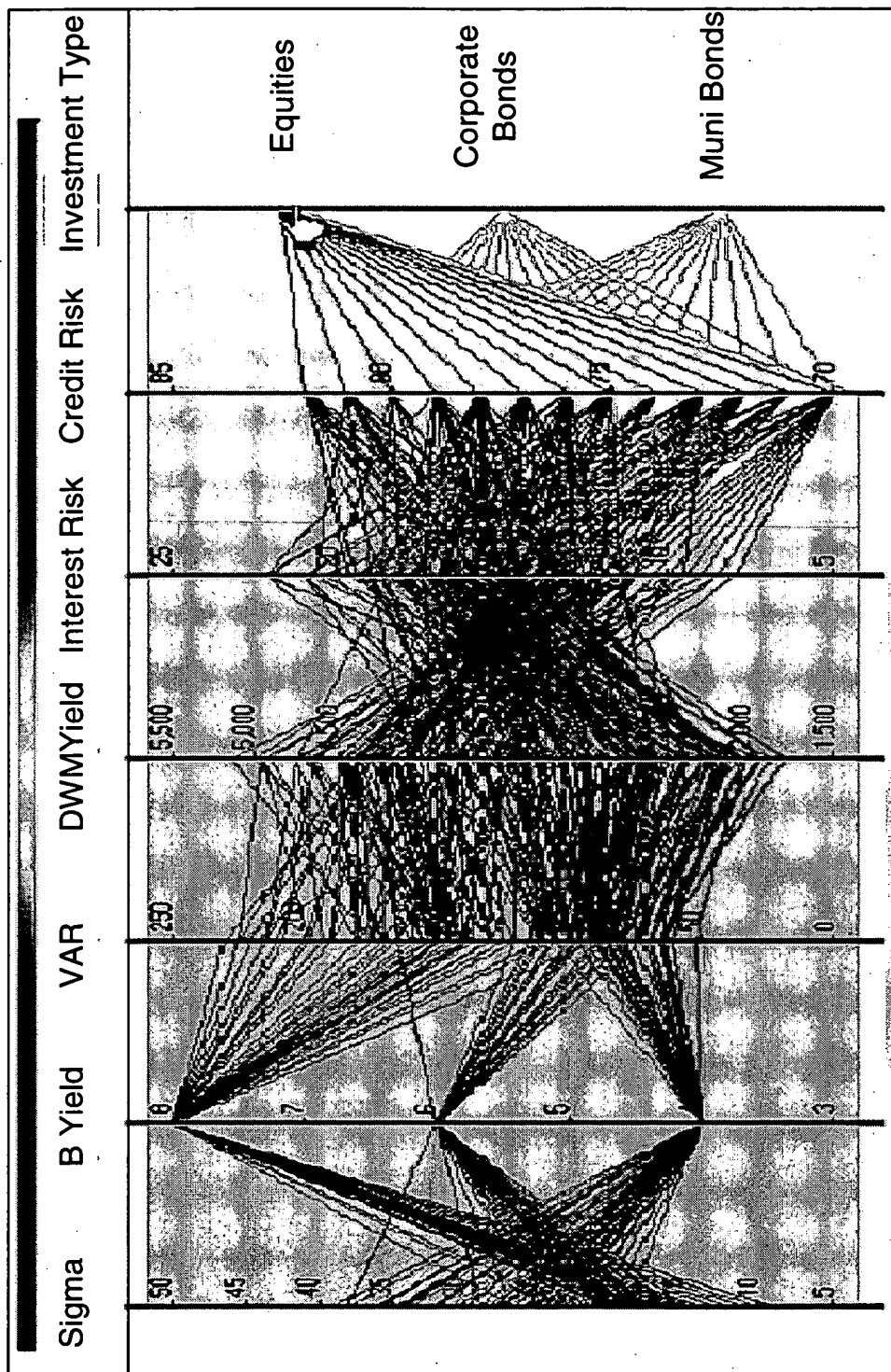


Fig. 14

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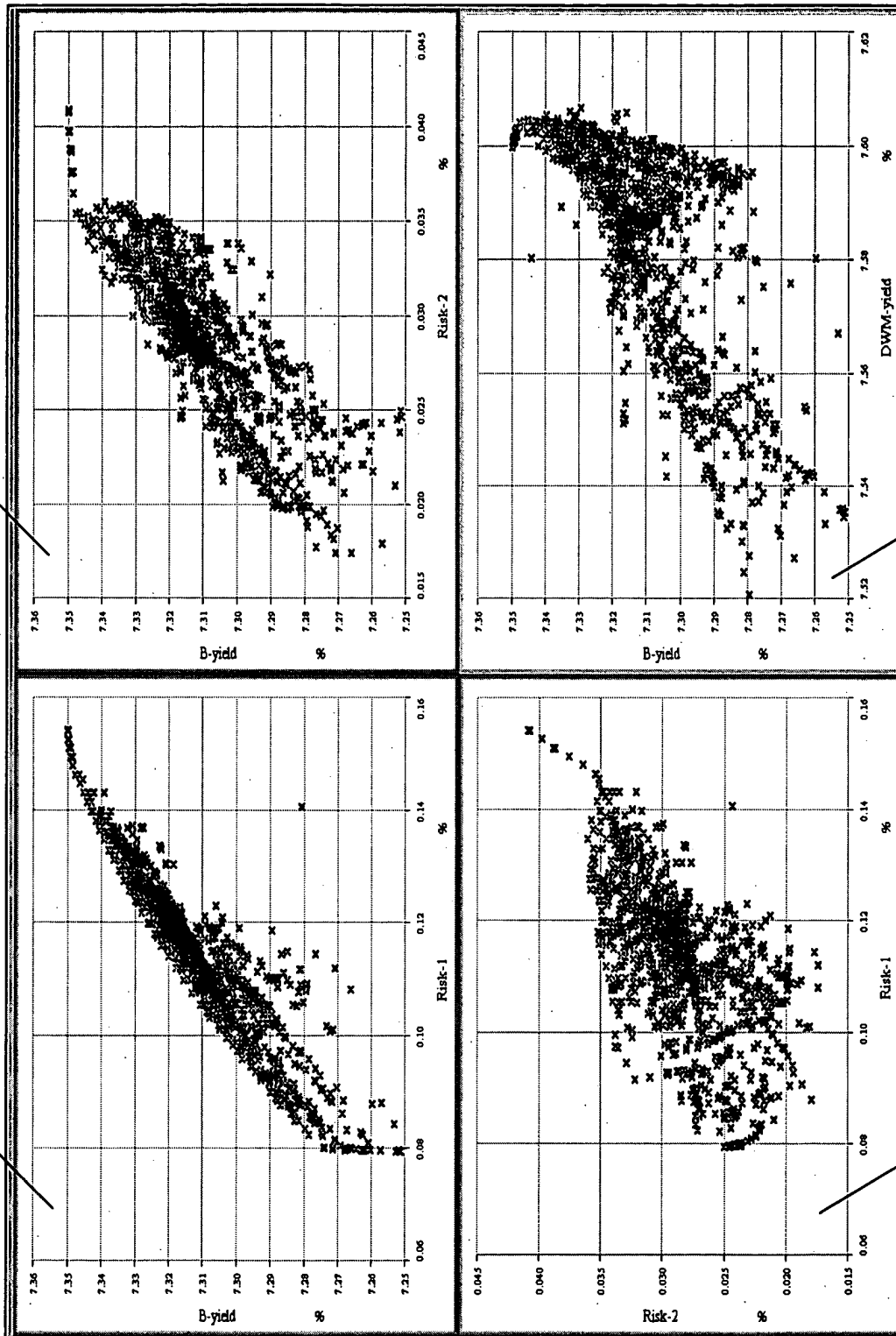


Fig. 15

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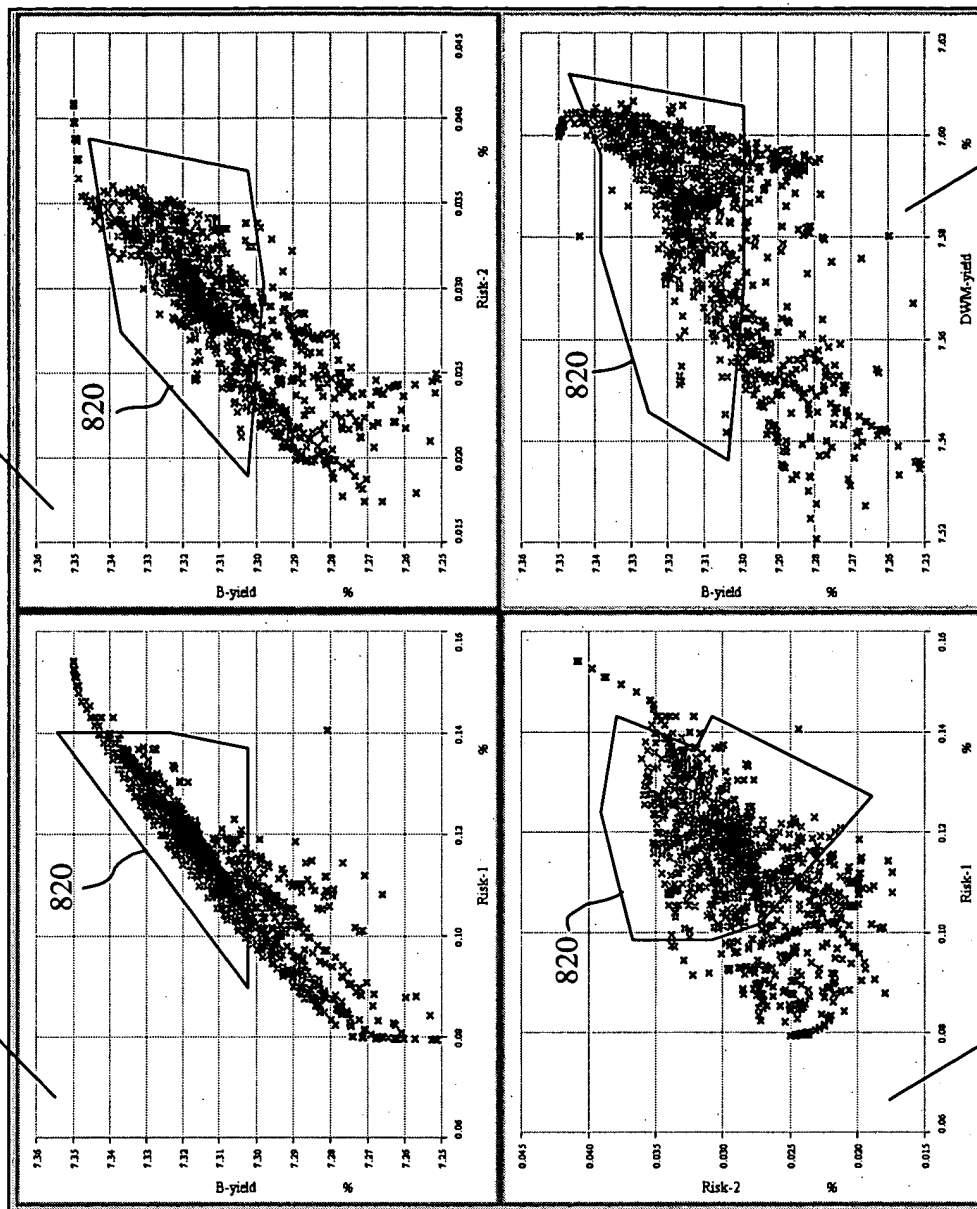


Fig. 16

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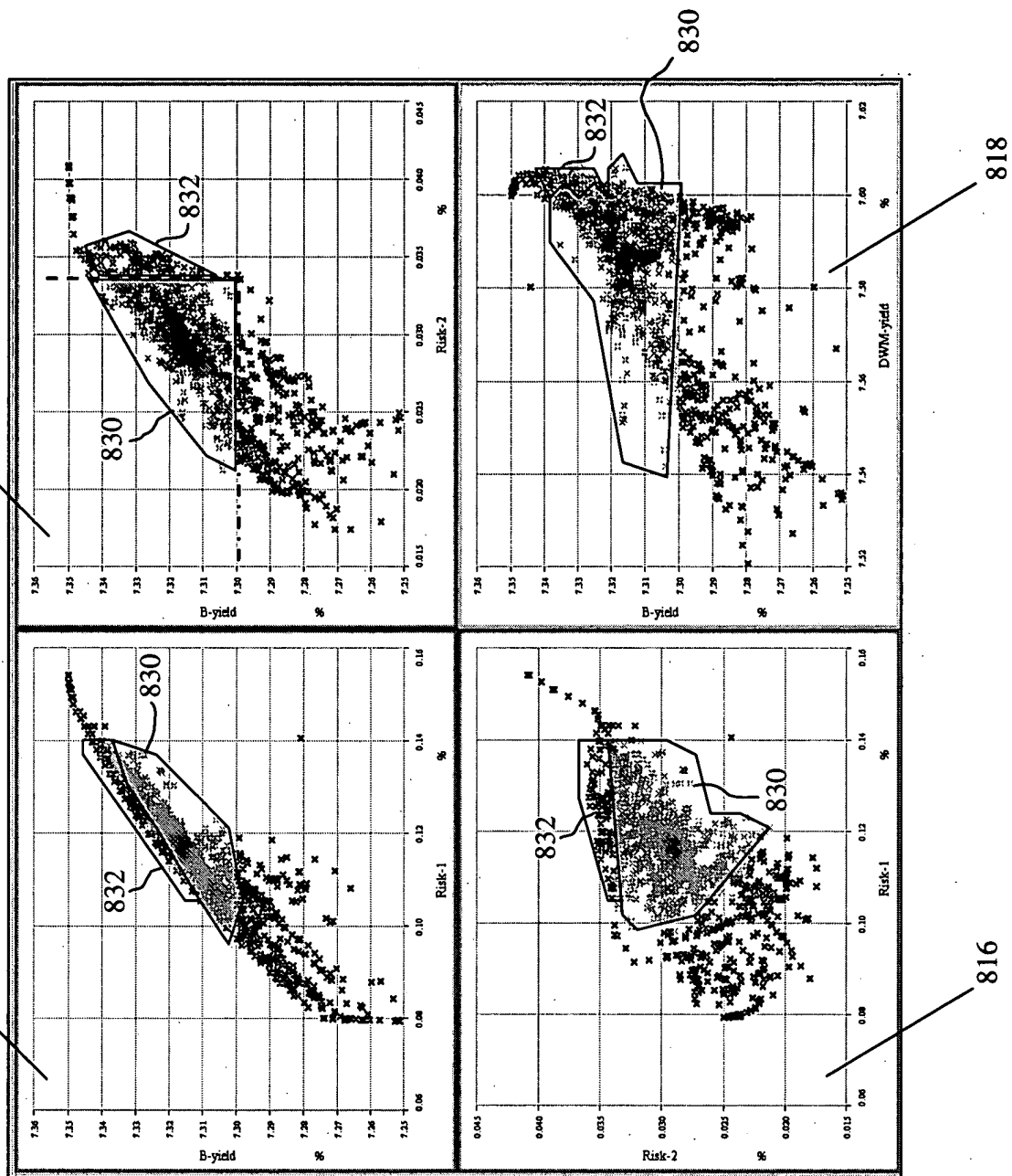
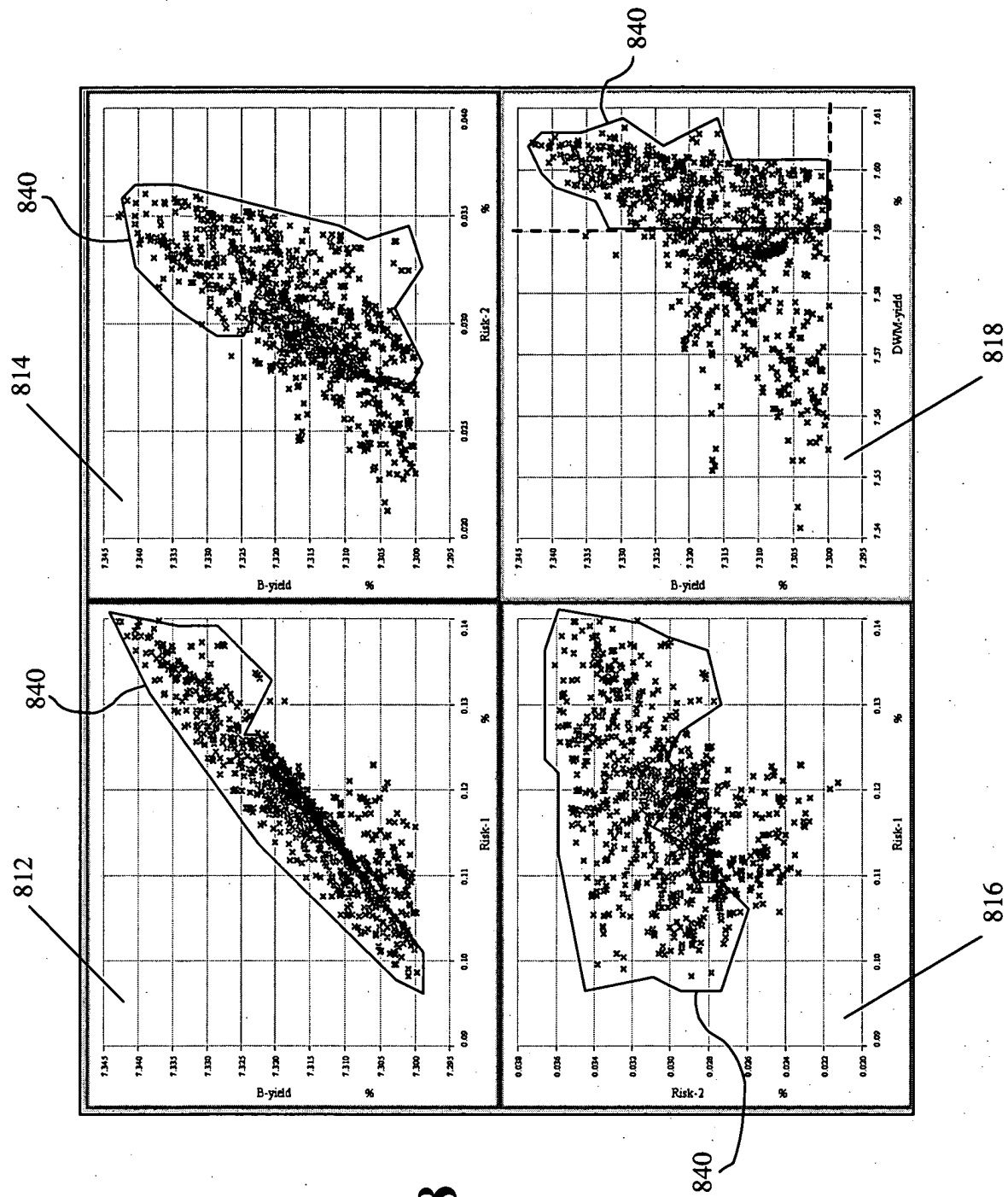


Fig. 17

Fig. 18



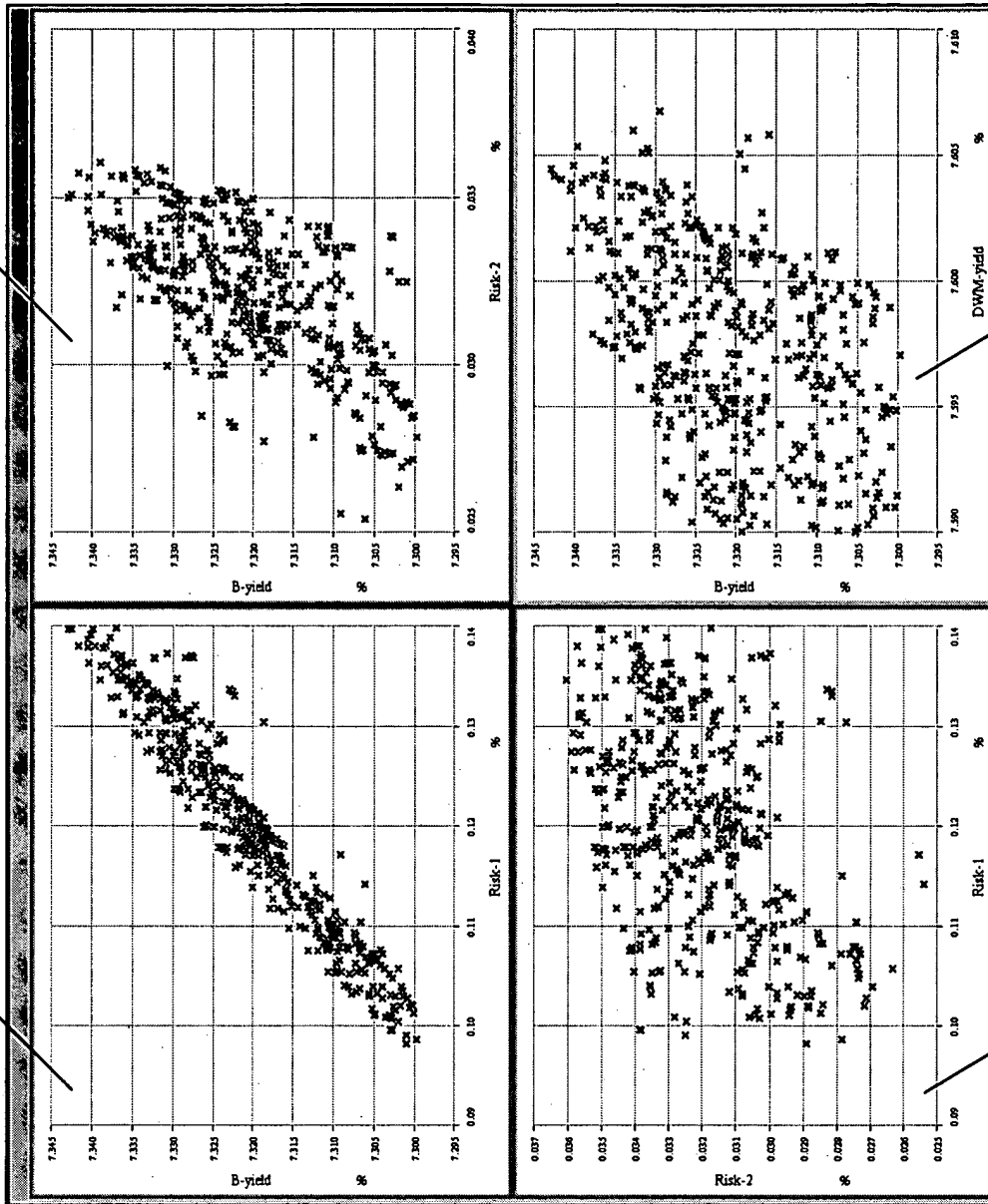


Fig. 19

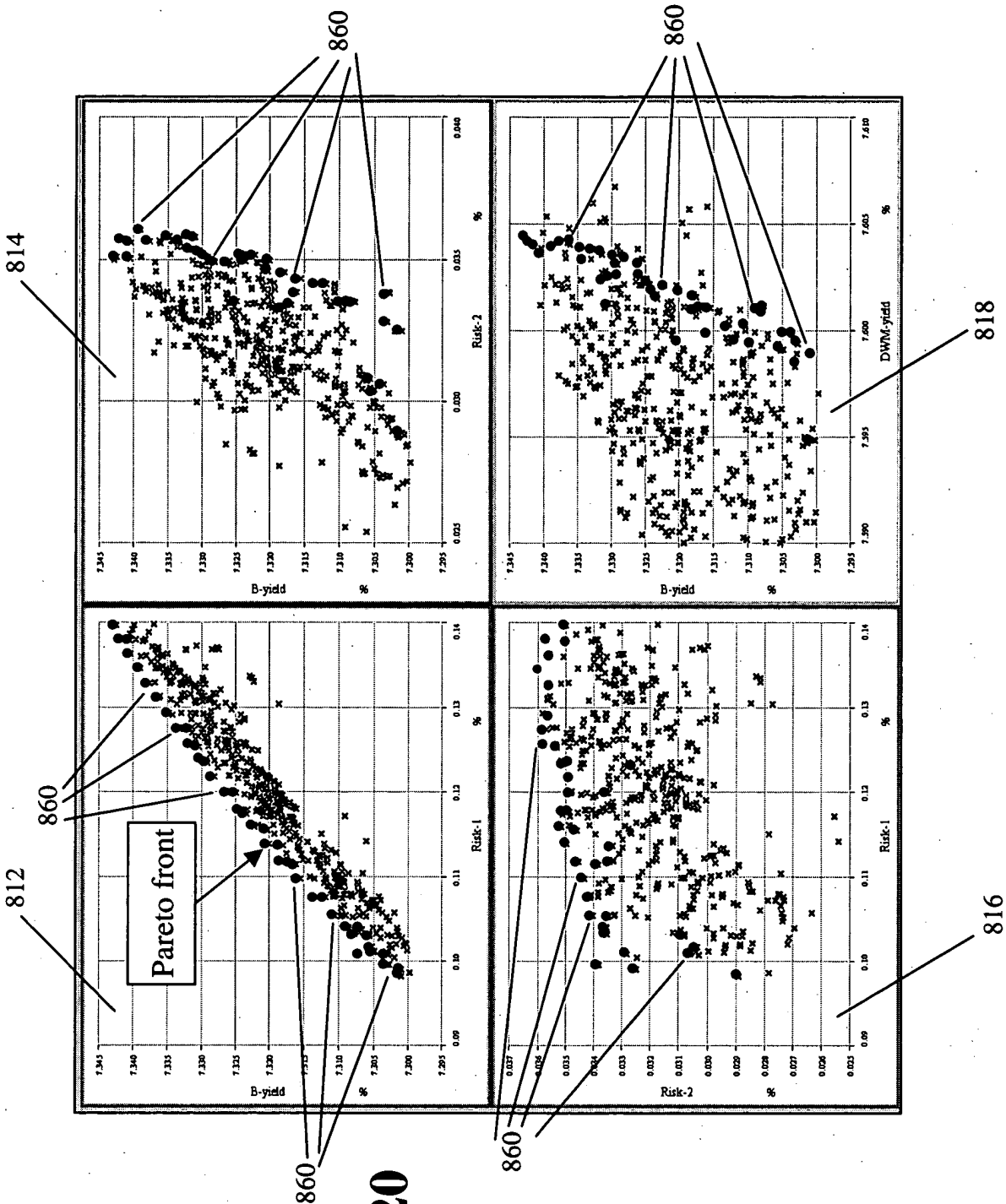


Fig. 20

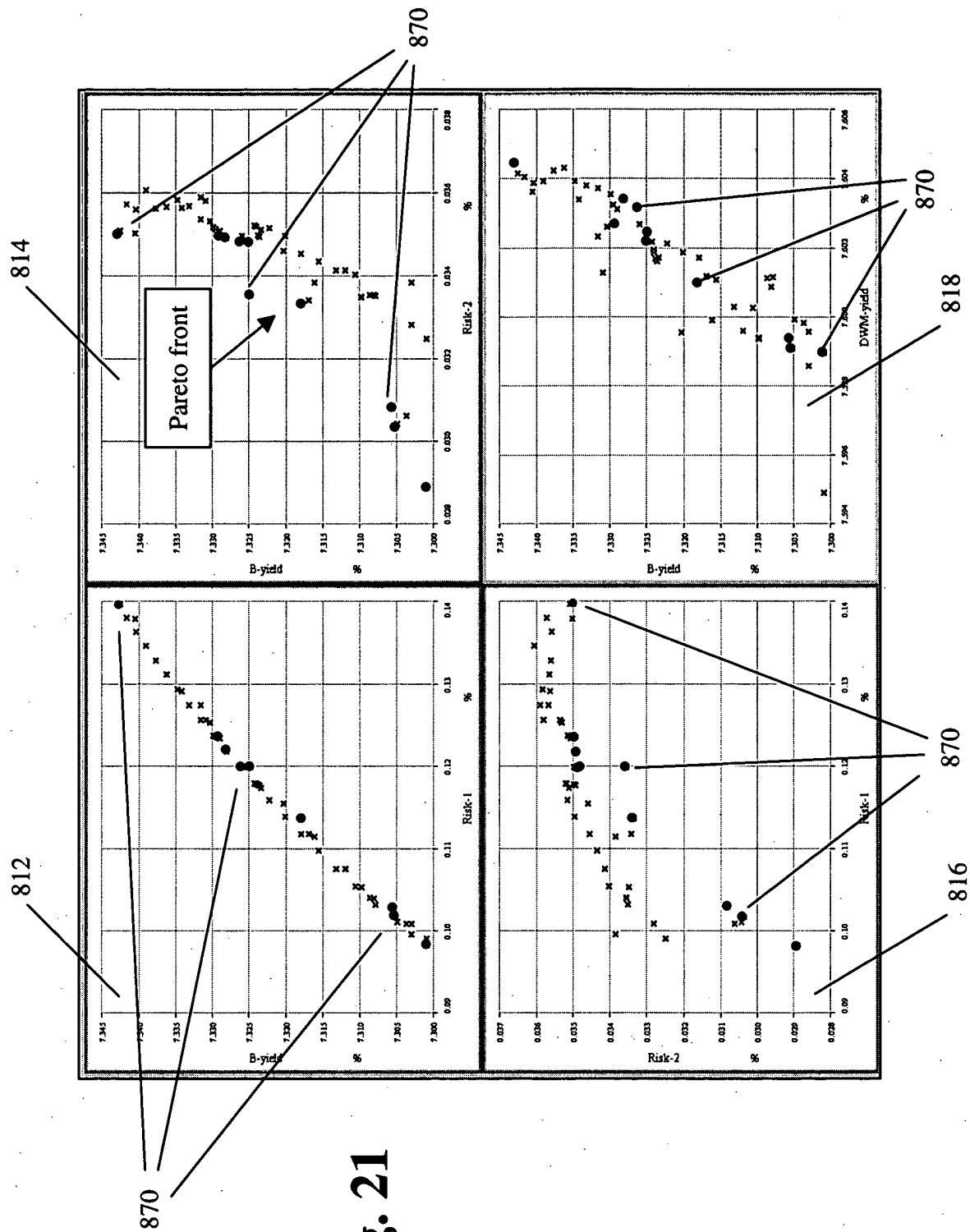
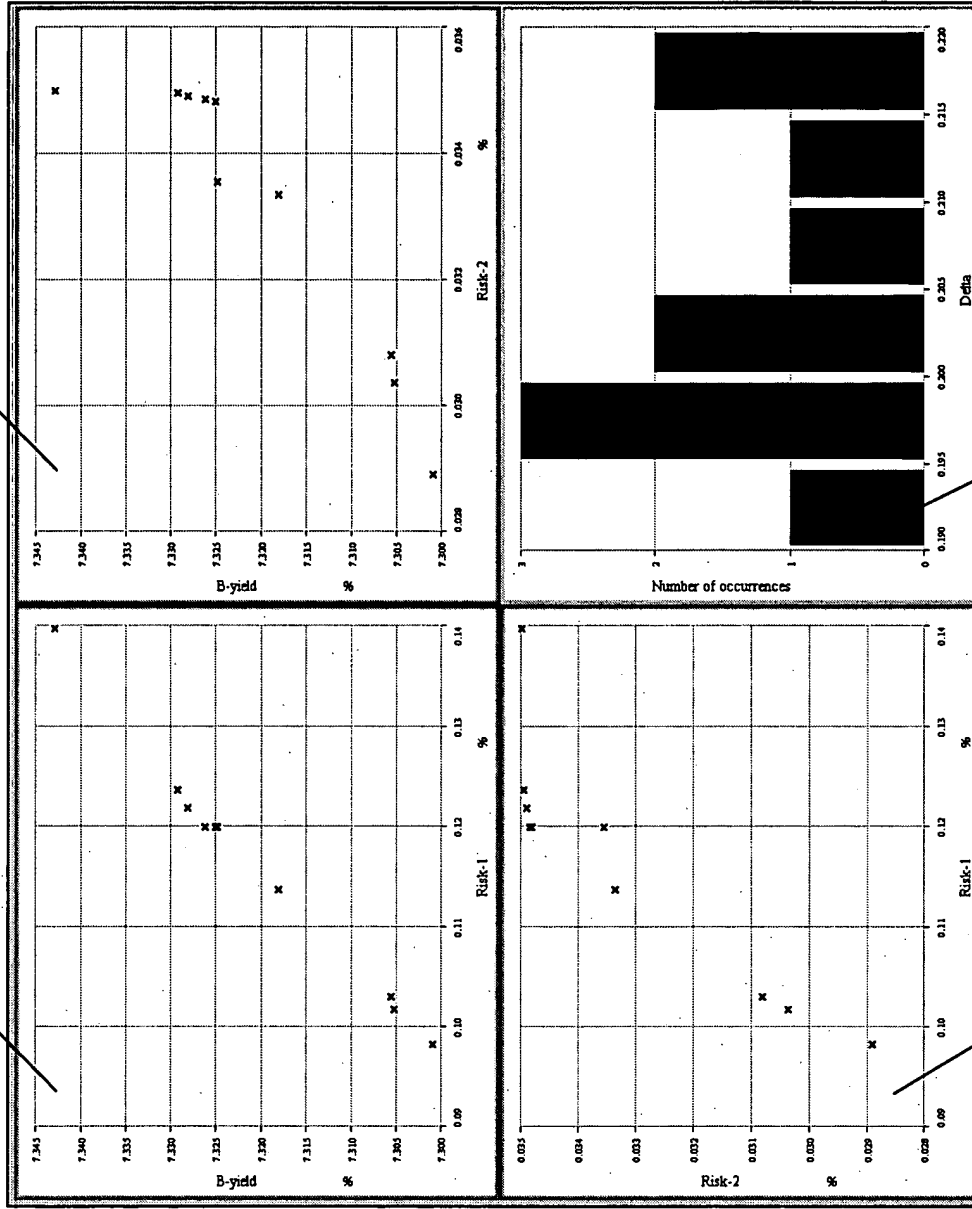


Fig. 21

Fig. 22



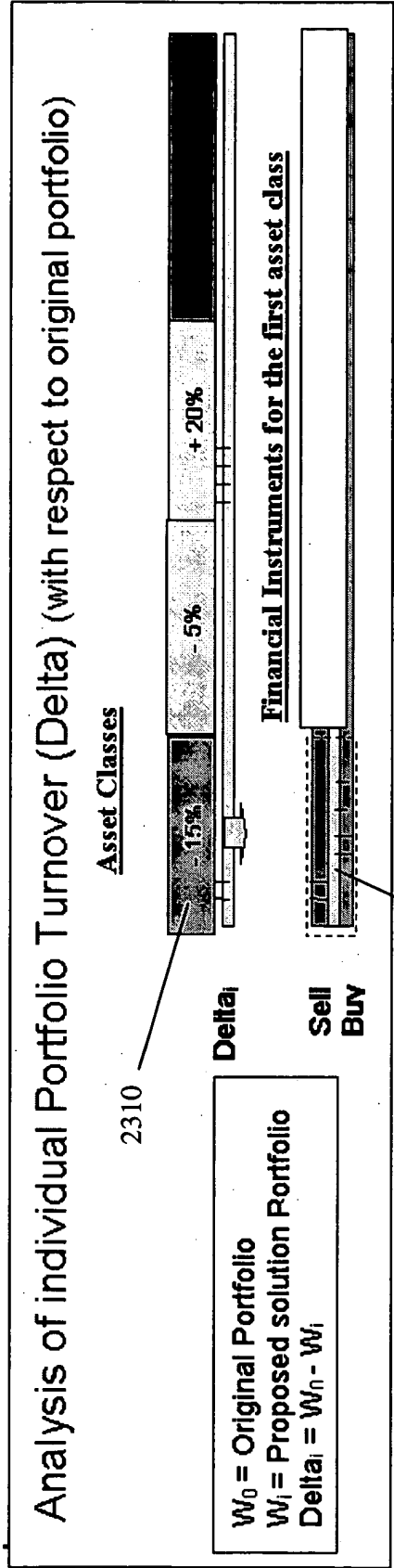


Fig. 23

Allocation	Asset Class 1	Asset Class 2	Asset Class 3	Asset Class 4	Asset Class 5	Total
Original Portfolio	35%	20%	5%	15%	25%	100%
P1	20%	15%	25%	15%	25%	100%
P2	40%	25%	10%	10%	15%	100%
P3	20%	20%	15%	20%	25%	100%
P4	15%	30%	20%	20%	15%	100%
P5	45%	20%	15%	10%	10%	100%
P6	20%	25%	20%	25%	10%	100%
P7	25%	25%	15%	20%	15%	100%
P8	30%	15%	10%	25%	20%	100%
P9	20%	25%	15%	20%	20%	100%
P10	30%	10%	15%	25%	20%	100%

Fig. 24

Deltas	Asset Class 1	Asset Class 2	Asset Class 3	Asset Class 4	Asset Class 5	Net Change
P1	-15%	-5%	20%	0%	0%	0%
P2	5%	5%	5%	-5%	-10%	0%
P3	-15%	0%	10%	5%	0%	0%
P4	-20%	10%	15%	5%	-10%	0%
P5	10%	0%	10%	-5%	-15%	0%
P6	-15%	5%	15%	10%	-15%	0%
P7	-10%	5%	10%	5%	-10%	0%
P8	-5%	-5%	5%	10%	-5%	0%
P9	-15%	5%	10%	5%	-5%	0%
P10	-5%	-10%	10%	10%	-5%	0%
Average	-9%	1%	11%	4%	-8%	
Median	-13%	3%	10%	5%	-8%	

Fig. 25

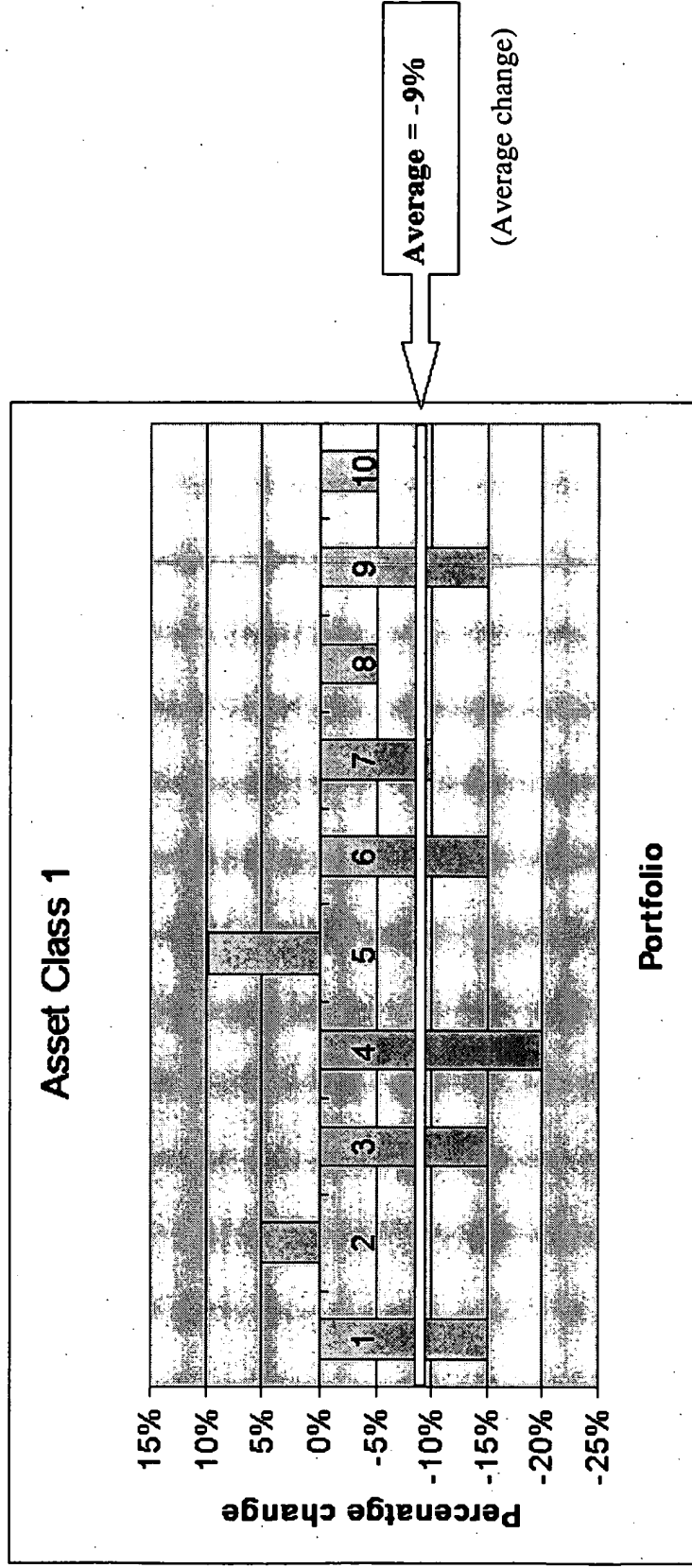


Fig. 26

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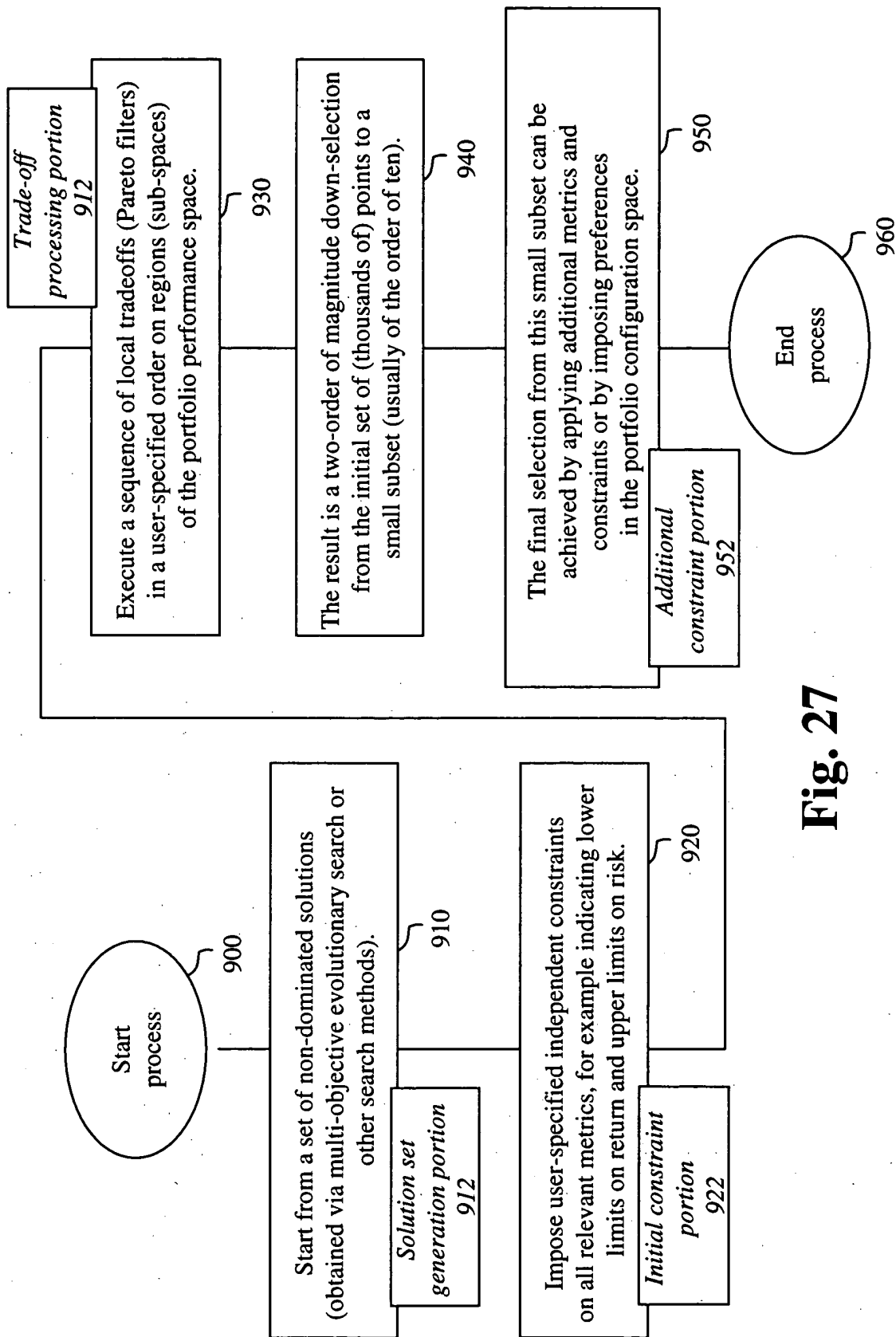


Fig. 27

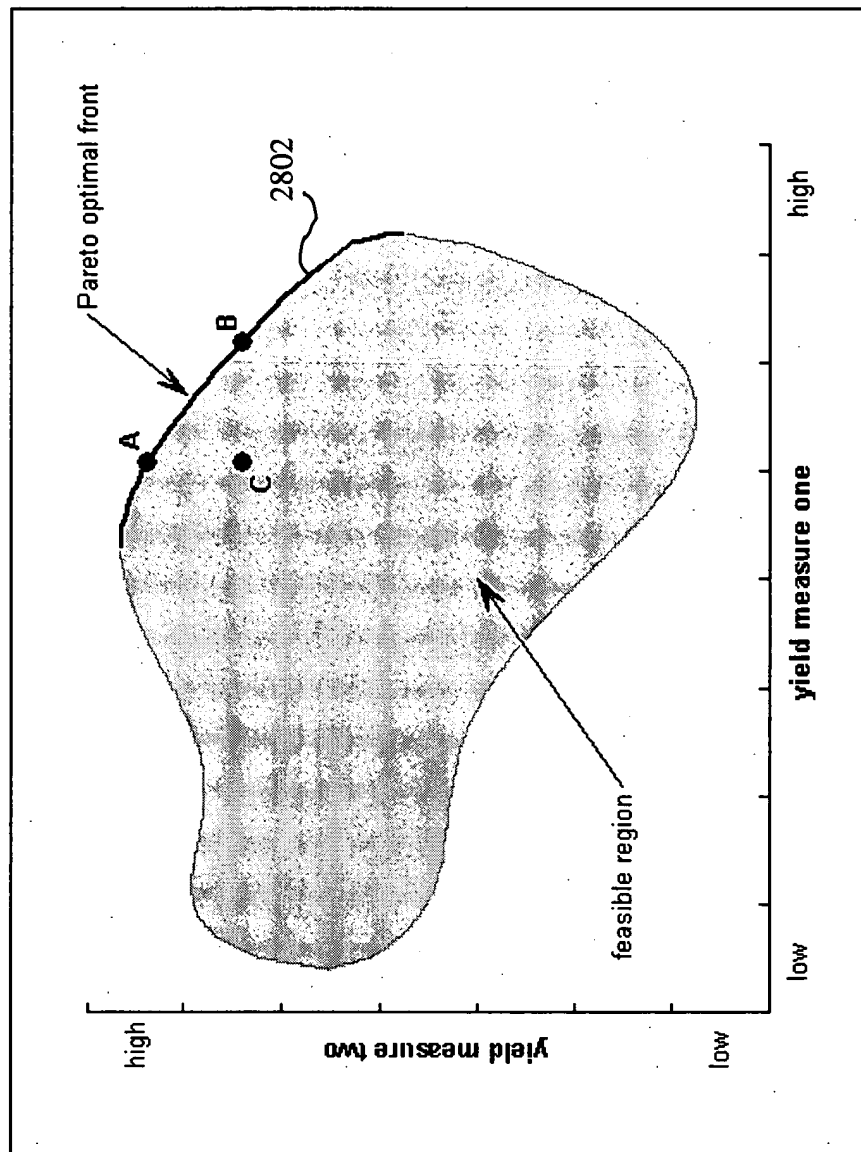


Fig. 28

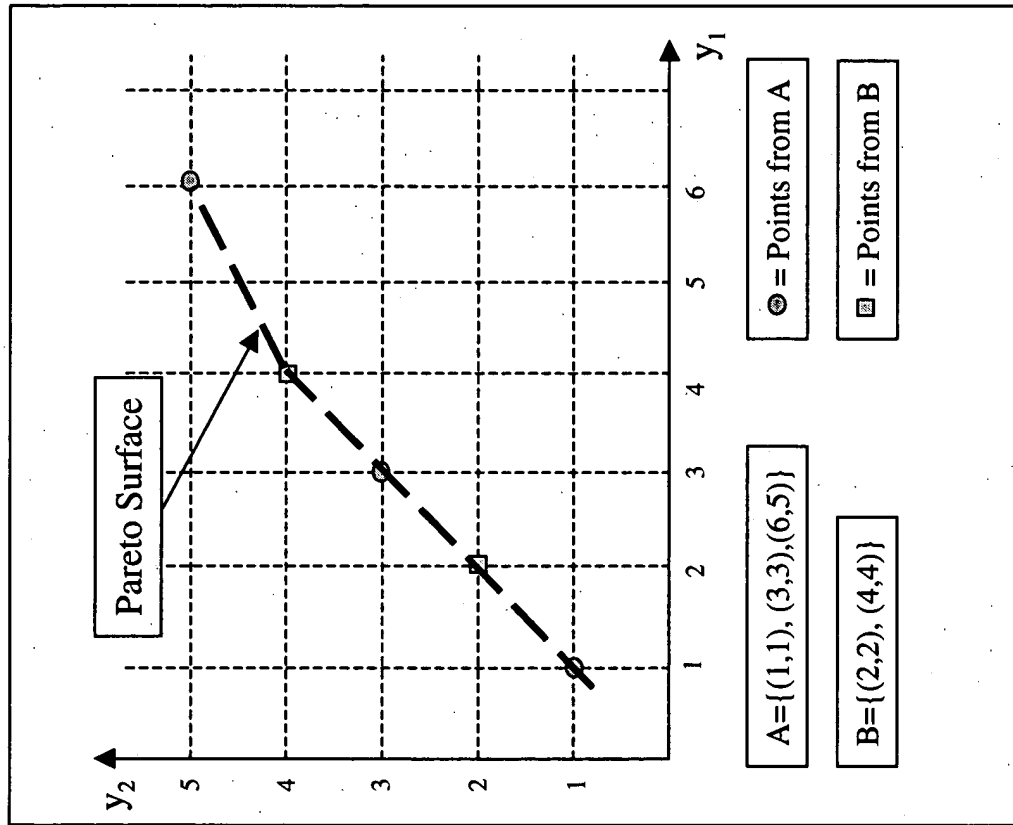
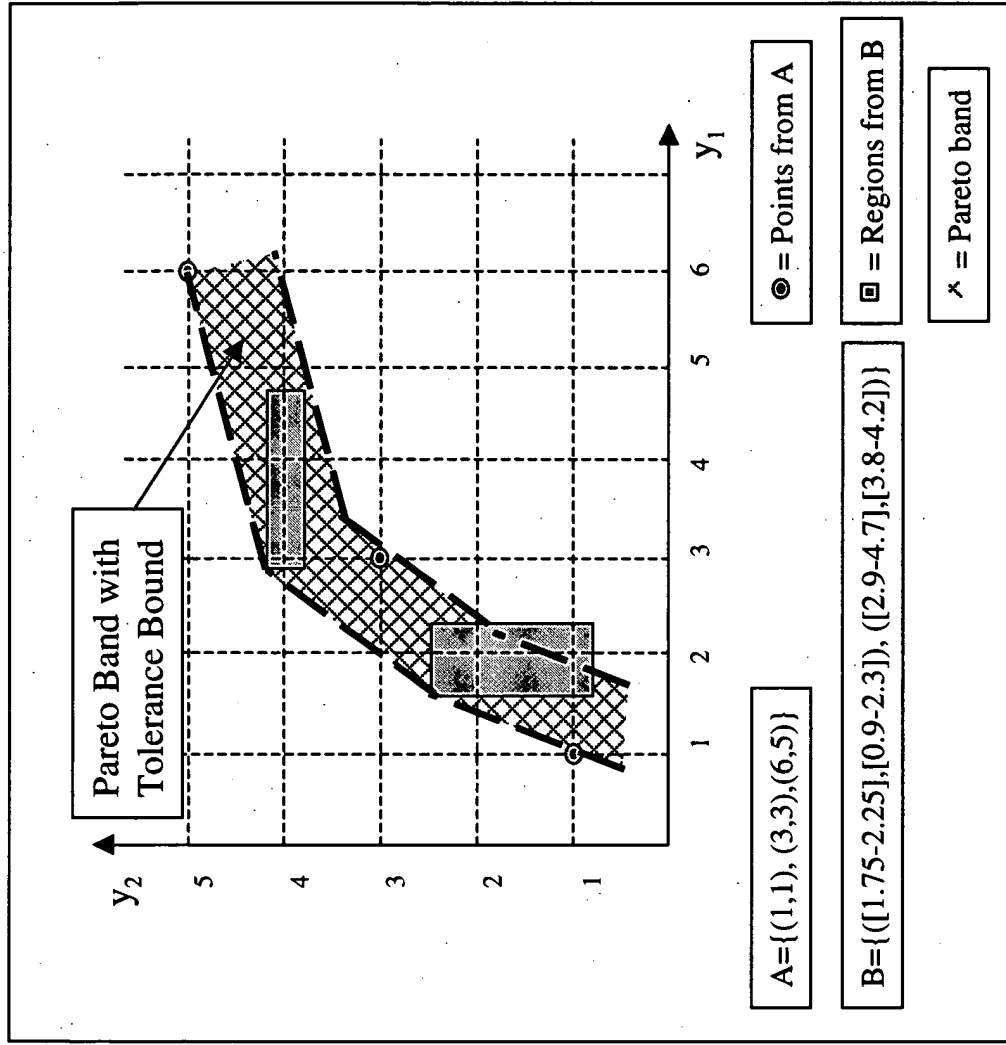


Figure 29

Deterministic Evaluation

Figure 30



Stochastic Evaluation (Transformed into Confidence Intervals)

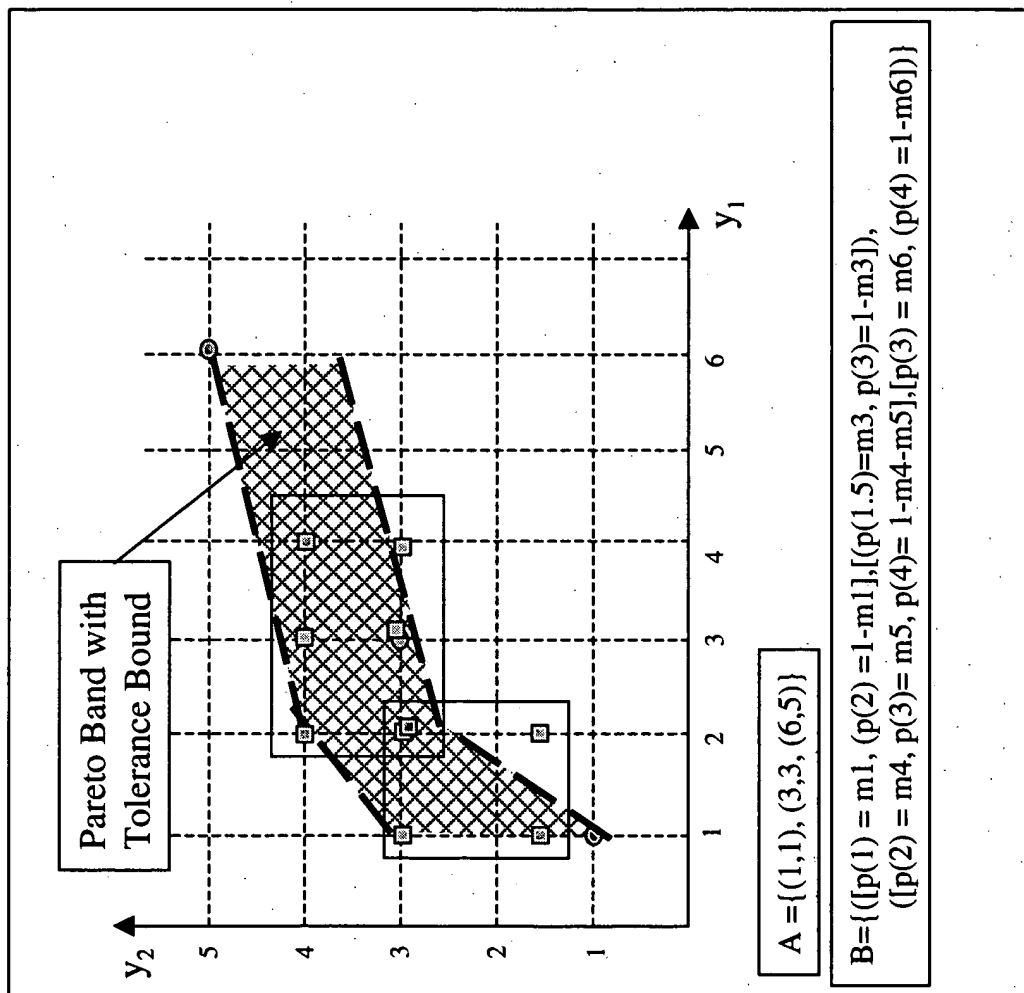
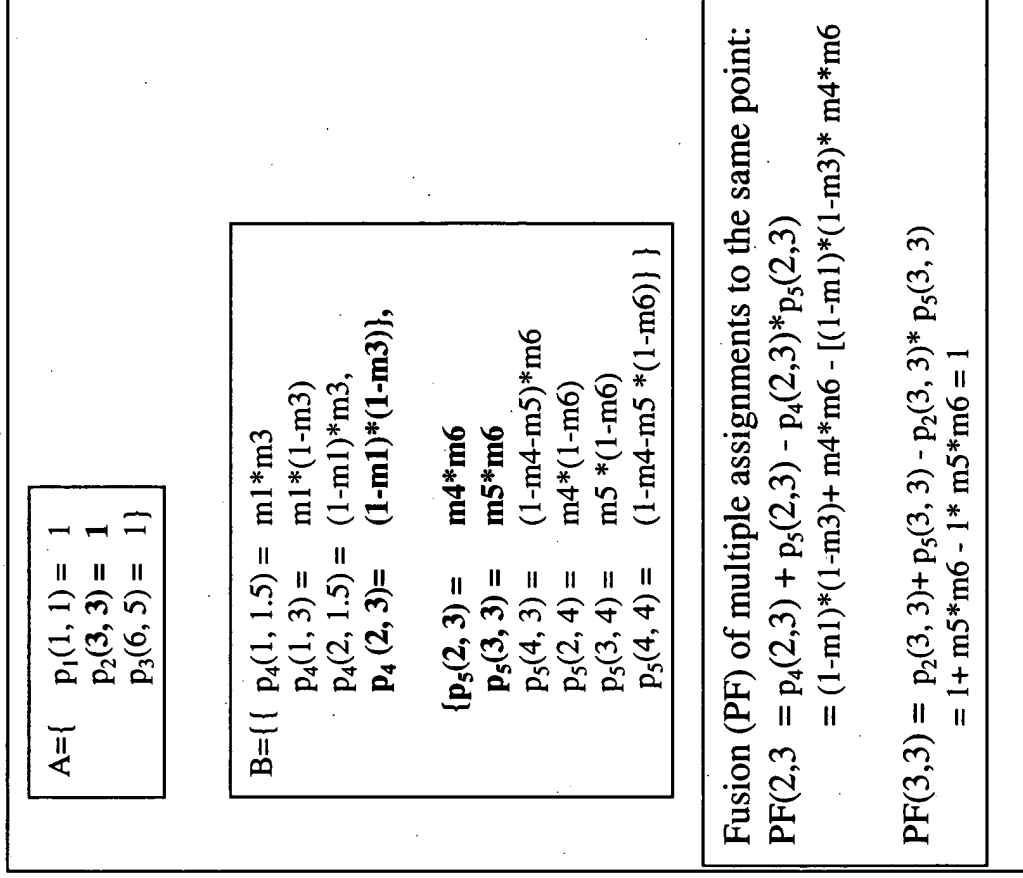


Figure 31

Discrete Probabilistic Evaluation

Figure 32



Probabilistic Fusion

Feasible Regions for Optimization

Figure 33


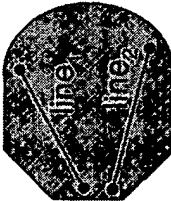
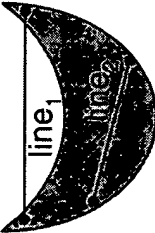
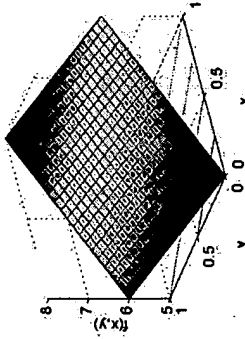
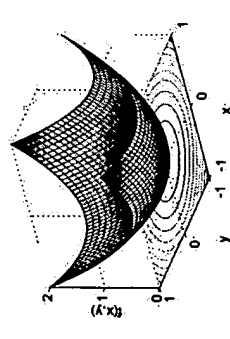
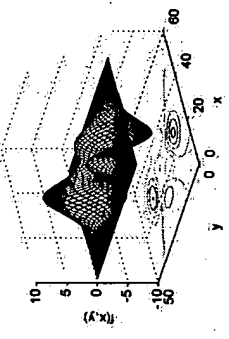
Graphic Visual	Word Description	Example Equation	GEAM
<p>Linear Convex Space</p> 	<ul style="list-style-type: none"> For any two points in the space, the line connecting the two points is always contained in the same space Space is defined using linear equations 	$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{81} & a_{82} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_8 \end{bmatrix}$ <p>Set of linear equations</p>	<ul style="list-style-type: none"> Market value weighted yield formulation Duration weighted yield formulation
<p>Nonlinear Convex Space</p> 	<ul style="list-style-type: none"> For any two points in the space, the line connecting the two points is always contained in the same space Space is defined using some nonlinear equations 	$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{51} & a_{52} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_5 \end{bmatrix}$ <p>Nonlinear equation</p> $x^2 + y^2 \leq \alpha$	<ul style="list-style-type: none"> Interest rate sigma formulation
<p>Nonlinear Nonconvex Space</p> 	<ul style="list-style-type: none"> For any two points in the space, the line connecting the two points is not always contained in the same space Space is defined using some nonlinear equations 	$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} x^2 \\ x \\ y \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ <p>Set of nonlinear equations</p>	<ul style="list-style-type: none"> Interest rate sigma and VAR formulation VAR is a nonlinear nonconvex constraint

Figure 34

Objective Functions

Graphic Visual	Word Description	Example Equation	GE/AM
<p>Linear Function</p> 	<ul style="list-style-type: none"> Function is defined using linear equations Straightforward math relationship Easy to optimize 	$f(x, y) = 2x + y + 5$	<ul style="list-style-type: none"> Market value weighted yield Duration weighted yield
<p>Nonlinear Convex Function</p> 	<ul style="list-style-type: none"> Function is defined using a nonlinear equation Functional gradients lead to single optimum Harder to optimize 	$f(x, y) = x^2 + y^2$	<ul style="list-style-type: none"> Interest rate sigma
<p>Nonlinear Nonconvex Function</p> 	<ul style="list-style-type: none"> Function is defined using complex nonlinear equations Multiple local optima Functional gradients are inefficient Very hard to optimize 	$f(x, y) = g_1(x, y) + g_2(x, y) + g_3(x, y) + g_4(x, y)$	<ul style="list-style-type: none"> Interest rate sigma and VAR

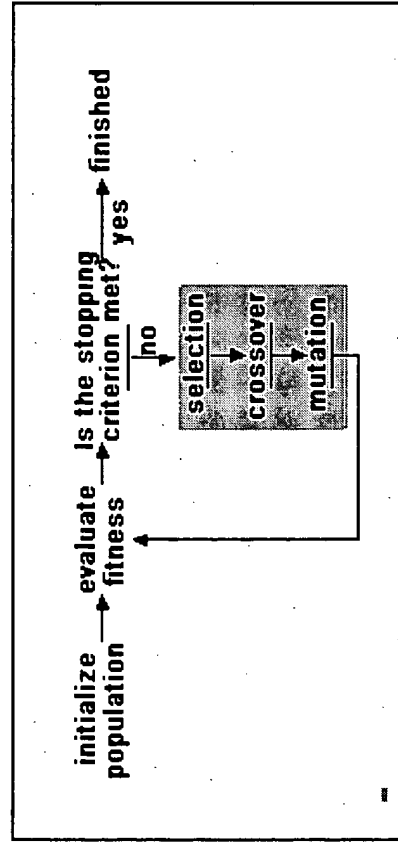


Figure 35

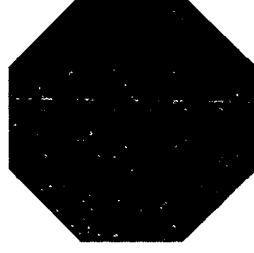
Evolutionary Search Augmented with Domain Knowledge

Multi-objective portfolio optimization problem is formulated as a problem with Multiple linear, nonlinear and nonlinear nonconvex objectives. However, the domain knowledge allows us to use strictly linear and convex constraints.

Knowledge about geometry of feasible space (i.e. convexity), allowed us develop a feasible space boundary sampling algorithm (solutions archive generation). By knowing the boundary of the search space, we can exploit that knowledge to design efficient interior sampling methods.

Convex crossover is a powerful interior sampling method, which is guaranteed to produce feasible offspring solutions. Given parents P_1 , P_2 , it creates offspring $O_1 = \lambda P_1 + (1 - \lambda)P_2$, $O_2 = (1 - \lambda)P_1 + \lambda P_2$. An offspring O_k and P_k can crossed over to produce more diverse offspring.

Linear Convex Feasible Space



Linear Convex Feasible Space

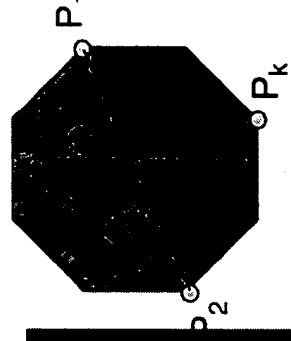
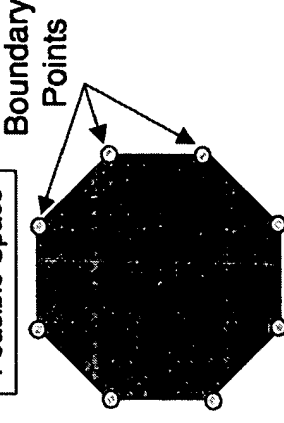


Figure
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Example of Outer Product using as operator the function $T(x,y)$

T-norm	Correlation Type
$T_1(x,y) = \max(0, x + y - 1)$	Extreme case of negative correlation
$T_2 = x * y$	No correlation
$T_3 = \min(x, y)$	Extreme case of positive correlation

Figure
37

Example of Outer Product using as operator the function $S(x,y)$

T-conorm	Correlation Type
$S_1 = \min(1, x + y)$	Extreme case of negative correlation
$S_2 = x + y - (x * y)$	No correlation
$S_3 = \max(x, y)$	Extreme case of positive correlation

Figure
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